

4/16/15

① See solutions from 4/14/15.

$$\textcircled{2} \quad \lim_{x \rightarrow 0^+} \cot(x)^{2x} = \lim_{x \rightarrow 0^+} e^{\ln(\cot x) \cdot 2x} = \exp \left\{ \lim_{x \rightarrow 0^+} 2x \ln(\cot x) \right\}$$

$$= \exp \left\{ 2 \lim_{x \rightarrow 0^+} \frac{\ln(\cot(x))}{\frac{1}{x}} \right\} \stackrel{\text{L'Hôpital}}{=} \exp \left\{ 2 \lim_{x \rightarrow 0^+} \frac{\frac{1}{\cot(x)} \cdot \csc^2(x)}{\frac{-1}{x^2}} \right\}$$

$$= \exp \left\{ 2 \lim_{x \rightarrow 0^+} \frac{-x^2 \sin(x)}{\cos(x) \sin^2(x)} \right\} = \exp \left\{ 2 \lim_{x \rightarrow 0^+} \frac{x}{\sin(x)} \lim_{x \rightarrow 0^+} \frac{-x}{\cos(x)} \right\}$$

$$= \exp \{ 2 \cdot 1 \cdot 0 \} = e^0 = \boxed{1}$$

③ (c) and (d).

$$\textcircled{4} \quad \frac{d}{dx} \left[ 2 \ln \left( \frac{x}{e^x + 1} \right) \right] = \frac{d}{dx} \left[ 2 \ln(x) - 2 \ln(e^x + 1) \right]$$

$$= \boxed{\frac{2}{x} - \frac{2}{e^x + 1} \cdot e^x}$$

$$\textcircled{5} \quad \int_a^b f(x) dx = \lim_{n \rightarrow \infty} \sum_{i=0}^{n-1} f\left(a + i \frac{b-a}{n}\right) \cdot \frac{b-a}{n}$$

$$\textcircled{6} \text{ a) } \int_0^{\pi/4} \sec^2 \theta d\theta = \tan \theta \Big|_0^{\pi/4} = \tan(\frac{\pi}{4}) - \tan(0) = \boxed{1}$$

$$\text{b) } \int_{-1}^1 |x| dx = \int_{-1}^0 |x| dx + \int_0^1 |x| dx$$

$$= \int_{-1}^0 -x dx + \int_0^1 x dx = \left[-\frac{1}{2}x^2\right]_{-1}^0 + \left[\frac{1}{2}x^2\right]_0^1$$

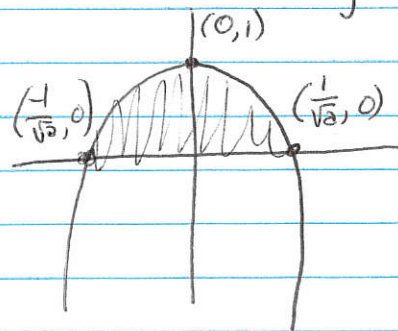
$$= \left[0 - \left(-\frac{1}{2}\right)\right] + \left[\frac{1}{2} - 0\right] = \frac{1}{2} + \frac{1}{2} = \boxed{1}$$

$$\text{c) } \int_2^4 \frac{2}{x} = 2 \ln(x) \Big|_2^4 = 2 \ln(4) - 2 \ln(2)$$

$$= 2 \ln\left(\frac{4}{2}\right) = \boxed{2 \ln(2)}$$

$$\textcircled{7} \quad 1 = \int_{-1/\sqrt{a}}^{1/\sqrt{a}} 1 - ax^2 dx$$

$$= \left[ x - \frac{a}{3}x^3 \right]_{-1/\sqrt{a}}^{1/\sqrt{a}}$$



$$y = 1 - ax^2$$

$$0 = 1 - ax^2$$

$$ax^2 = 1$$

$$x^2 = \frac{1}{a}$$

$$x = \pm \frac{1}{\sqrt{a}}$$

$$= \left[ \frac{1}{\sqrt{a}} - \frac{a}{3} \left(\frac{1}{\sqrt{a}}\right)^3 \right] - \left[ -\frac{1}{\sqrt{a}} - \frac{a}{3} \left(-\frac{1}{\sqrt{a}}\right)^3 \right]$$

$$= \frac{2}{\sqrt{a}} - \frac{2a}{3\sqrt{a}} = \frac{4}{3\sqrt{a}} \Rightarrow 3\sqrt{a} = 4$$

$$\Rightarrow \sqrt{a} = \frac{4}{3} \Rightarrow \boxed{a = \frac{16}{9}}$$